**Noninteracting GF’s**

Let’s examine a bunch of non-interacting GF’s. Note I’m going to be presuming time-development starts at t = 0: so U0(t) = e-iH0·t. But if we used U0(t,t0) = e-iH0(t-t0) instead, this wouldn’t change G(t,t´) because it is, as you’ll see, a function only of the difference of the arguments. Also, for the most part, I won’t bother with the exponential convergence factors that we should have on the real space GF’s.

**Elastic Field**

The elastic field Lagrangian is:



We’ll start with the causal GF.



where the expectation is taken with respect to the empty state, which is the G.S. too in this case. We can insert the FFE into this expression



And so we’ll get:



where in the second line we’ve kept the only two non-zero terms. Then using the commutation relations:



Can write this as, splitting into two terms, and changing **k** → -**k** in the second one:



If we had included the implicit convergence factor that should be attached to all of those θ functions (as aforementioned in the Formal Properties file), we would’ve gotten:



where ε = 0+. Now consider following identity, using the residue theorem,



So we can write:



and so the spatio-temporal Fourier transform would be, with help from implicit convergence factor (see single particle GF’s for more explicit use):



ρ being mass density of course. Now if we didn’t have an isotropic Hamiltonian here, then the eigenfrequencies/energies ωk would’ve also depended on λ → so ωkλ. And the polarization vectors may also have. But such is not the case for us, and so the sum over polarization vectors will just decouple and we’ll get:



We can get other GF’s. For instance consider the retarded GF:



where we changed **k** → -**k** in the second sum. If we had employed the convergence factor implicitly attached to the θ function, we would’ve obtained,



We’re ultimately interested in the Fourier transform. So let’s integrate over time against eiω(t-t´).



And it is clear that the total Fourier transform is then, presuming spectral independence from the polarization vectors:



We could get more, but I guess I won’t.

**Real Boson Field**

The real boson field Lagrangian is:



and the φ FFE is:



Now let’s look at the causal Green’s function.



where the expectation is taken with respect to the empty state, which is the G.S. too in this case. We can insert the FFE into this expression



where in the second line we’ve kept the only two non-zero terms,



Can write this as, splitting into two terms, and changing **k** → -**k** in the second one:



If we had included the convergence factor implicitly attached to the θ’s, we would’ve found,



Now consider following identity, using the residue theorem,



So we can write:



and finally, understanding kμ to be (k0, **k**), and therefore k2 to be k02 - |**k**|2, we have:



and by the way,



where we understand k2 = ω2 - |**k**|2. I guess I won’t bother with the other Green’s functions. This one seems to be all we need.

**Complex Boson Field**

The complex boson field Lagrangian is:



and the φ FFE is:



Now let’s look at the causal Green’s function for the complex boson.



where the expectation is taken with respect to the empty state, which is the G.S. too in this case. We can insert the FFE into this expression



where in the second line we’ve kept the only two non-zero terms. The rest will be as before, and so we’ll just get:



and,



**Free Fermion Field**

The fermion field Lagrangian,



and the FFE is:



Now let’s look at the fermion Green’s function.



where the expectation is taken with respect to the empty state, which is the G.S. too in this case. We can insert the FFE into this expression



where in the second line we’ve kept the only two non-zero terms and note the negative sign. That’s because transposing fermion operators comes at a negative sign cost, just like in the 2nd quantization formalism. And then using the identity:



we have:



The expression in brackets we’ve already evaluated above. And so we can say:



Sliding the operator back in, we have:



and also,



But there is a shorthand way to write this. Noting that



we can write these as:



and,



But these forms are more like mnemonics, because we have matrices in the denominator which isn’t really kosher.

**Free Photon Field**

And the photon Lagrangian is [using Natural Lorentz Heaviside units]:



And the FFE is:



And let’s do the free photon field Green’s function:



In the Coulomb gauge, this works out too:



We need to separate out the time part, to put it in the form of an integral over dk(0). So in the second term make the substitution **k** → -**k**, but recall that **ε**(-**k**,λ) → -**ε**(**k**,λ), and so the product of the two together won’t actually change. So then,



and so we finally have, changing notation k0 → ω:



and,



Consider the completeness relationship:



where is the unit vector pointing along the vector **k** direction, we might write this alternatively as:



If we were to go to the continuum, then we’d have:



where k2 = (k0)2 - |**k**|2 = ω2 - |**k**|2 as usual. Now let’s do the calculation in the Lorentz gauge. The Lagrangian was:



The FFE is:



and we’re still interested in:



which is now (using ξ = 1),



at this point we can see where this is going, and we’ll have:



Now using the identity from the free-photon file:



we have:



Finally, in Condensed Matter Physics, it is more common to use ‘Natural Gaussian’ units than the ‘Natural Lorentz Heaviside’ units we’ve been working in. And to make contact with that, when it comes, it’d be nice to see what the GF’s look like in *those* fake units. One way is to simply redo the derivation in those units. Another way is to convert our Natural Lorentz Heaviside expression to SI and then to Natural Gaussian. If we were to convert to SI, then we’d need our position-space photon GF,



to come out to units of magnetic vector potential squared, ‘twould seem. So to start, we need the denominator of that fraction to all have the same units. We can effectuate this by adding in a factor of c to **k**, which is kosher because c has magnitude 1 in Natural Lorentz Heaviside units so we haven’t changed anything. The iε we can ignore. And then we’d just tack on factors of ℏ, ε0, and µ0. So we need,



where units of vector potential, in SI, are [A] = [ML/CT]. And this requires:



Plugging the fourth into the second gives 2 = p – (q-r) = p – (-1) → p = 1. So then q = -1. And r = 0. So we should have, in SI:



And then if we were to convert to the fake ‘Natural Gaussian’ units, we’d set 4πε0 = 1, µ0/4π = 1 (which makes c = 1 again) and ℏ = 1. This gives us:

